

Analysis of Carrier Frequency Errors and Their Effects on OFDM Based Systems

Anil M. Jadhao¹, Dr. Ashok A. Ghatol²

Department of Applied Electronics, Sant Gadge Baba Amravati University, Amravati¹

Ex-Vice Chancellor, Dr. Babasaheb Ambedkar Technological University, Lonere²

Email:ajadhao@gmail.com¹, vc_2005@rediffmail.com²

Abstract- Orthogonal frequency division multiplexing (OFDM) is being used successfully in various applications. It was chosen for IEEE 802.11a wireless local area network (WLAN) standard, and it is being considered for the fourth- generation mobile communication systems. Along with its many attractive features, OFDM has some principal drawbacks. Before an OFDM receiver demodulates the sub-carriers, it performs two synchronization tasks. First, it determines the symbol boundaries and the optimal timing instants in order to minimize both the ICI and the ISI. Second, it tries to estimate and correct the frequency errors. OFDM's sensitivity to these frequency errors is one of its main drawbacks. This paper discusses the effects of these frequency errors on OFDM systems.

Index Terms- AWGN; BER; FFT; Frequency offset; ICI; IFFT; ISI; OFDM; Phase Noise.

1. INTRODUCTION

The orthogonality of the sub-carriers can be ensured only if the receiver and the transmitter have the same reference frequency. Any deviation from this reference frequency may cause ICI and loss of orthogonality. Another frequency error factor is phase noise, which is caused by random jitter of the phase of the steady sinusoidal waveform generated by the

receiver and the transmitter, and phase noise introduced by non- linear channels.

Figure 1 shows the front end of an OFDM receiver where most of the frequency errors occur, i.e., the local oscillators and the sample clock at the analog to digital (A/D) converter [1]. The A/D converter causes errors when the receiver does not have the same sample clock frequency as at the transmitter.

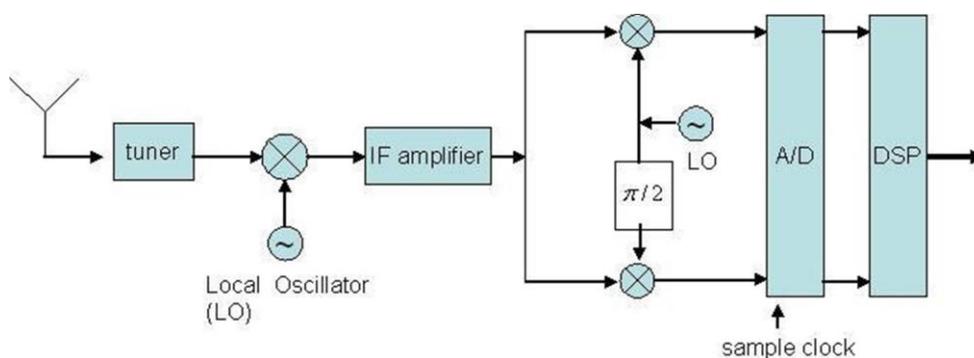


Figure 1: OFDM Receiver Front End

oscillators [5].

Typically frequency errors are generated by the fact that the oscillators in the modulator and demodulator do not have exactly the same frequency.

For single-carrier systems, the effect of phase noise and frequency offset appear only as degradation in the received SNR, rather than ISI or ICI. Nevertheless, many efficient techniques to minimize the effects of this drawback have been proposed in the literature [6, 7].

Other reasons for frequency errors include Doppler shift caused by the relative movement between the

The frequency offset and the phase noise cause a phase rotation of the received symbols. Coherent OFDM systems need a phase tracking device to obtain the phase of the incoming symbols for correct demodulation [7]. Typically, three types of algorithms are used to estimate the frequency offsets, i.e., to track the phase, in coherent OFDM systems:

- data-aided algorithms that use special training information (pilot tones) within the transmitted signal,
- non-data-aided algorithms that analyze the received signal in the frequency domain, and

- algorithms that use the cyclic prefix of the OFDM signals [6].

For wireless applications, the first method is the method of choice. The implementation details of these coherent OFDM detection algorithms can be found in [6]. It should be noted that in many applications as well as in this paper, the differential detection technique is used since it does not require channel estimation and the use of pilot tones, even though it causes a loss of SNR.

In the following sections, the effects of each of the aforementioned reasons that cause frequency errors are analyzed. A simple representation of an OFDM signal is created for the ease of analysis.

2. CREATING A SIMPLE REPRESENTATION OF AN OFDM SIGNAL TO SIMPLIFY ANALYSIS

A baseband OFDM signal can be represented by [4]

$$b(t) = \sum_{i=1}^{N-1} A_i \cos(w_i t + \phi_i) \quad (1)$$

Where A_i is the amplitude, $w_i=2\pi f_i$ is the angular frequency, ϕ_i is the phase of the i^{th} sub-carrier, and N is the number of sub-carriers.

According to the *modulation* technique to be used, either A or f is determined by the data, the baseband OFDM signal $b(t)$ is modulated next, onto a RF carrier with frequency f_c :

$$s(t) = 2b(t) \cos(w_c t)$$

$$= 2 \sum_{i=0}^{N-1} A_i \cos(w_i t + \phi_i) \cos(w_c t) \quad (6)$$

$$= \sum_{i=0}^{N-1} A_i \{ \cos[(w_c + w_i)t + \phi_i] + \cos[(w_c - w_i)t - \phi_i] \} \quad (2)$$

where $w_c = 2\pi f_c$, and we assume the phase of the carrier to be zero for simplicity. Since a single side band transmission is enough to carry the information in A_i or ϕ_i ,

it is assumed that the upper sideband is used, and therefore the transmitted signal can be represented as [4]

$$s(t) = \sum_{i=0}^{N-1} A_i \cos[(w_c + w_i)t + \phi_i] \quad (3)$$

3. THE EFFECT OF DOPPLER FREQUENCY SHIFT, PHASE NOISE AND OSCILLATOR FREQUENCY OFFSET ON THE OFDM SYSTEMS

In this section the theoretical analysis of the effects of frequency errors is presented. The maximum Doppler shift occurs when the two mobile nodes move toward each other, given by [8]

$$f_d = \frac{v f_c}{c} \quad (4)$$

where v is the relative speed of the two nodes, f_c is the carrier frequency and c is the speed of light (3×10^8 m/s).

3.1. Effect of Doppler Shift on the Carrier Frequencies, Sub-carriers, Envelope and Symbol Timing

An OFDM signal consists of numerous sub-carriers with different frequencies.

The amount of Doppler shift affecting the i^{th} sub-carrier is given by [4]

$$(f_c - f_i) \xrightarrow{\text{Doppler Shift}} (1 + \xi)(f_c \pm f_i) \quad (5)$$

Where ξ is the percentage of the change in frequency and is determined by

$$\xi = \frac{f_d}{f} = \frac{v}{c} \cos \theta$$

The right-hand side of Equation 5 can be written as $(1 + \xi)(f_c \pm f_i) = (1 + \xi)f_c \pm (1 + \xi)f_i$ (7)

which demonstrates that the Doppler frequency shift affects the carrier frequency and the sub-carrier frequencies by the same percentage ξ [4]. The Doppler shift of the carrier frequency can be calculated as

$$f_{dc} = \frac{v f_c}{c} \cos \theta \quad (8)$$

and the Doppler shift of the sub-carrier frequencies as

$$f_{di} = \frac{v f_i}{c} \cos \theta \quad (9)$$

By using Equation 3 again, the transmitted OFDM signal with Doppler shift can be written as

$$s(t) = \sum_{i=0}^{N-1} A_i \cos[(1 + \xi)(w_c + w_i)t + \phi_i]$$

$$= \sum_{i=0}^{N-1} \{A_i \cos[(1 + \xi)w_i t + \phi_i] \cos[(1 + \xi)w_c t] - A_i \sin[(1 + \xi)w_i t + \phi_i] \sin[(1 + \xi)w_c t]\}$$

(10)

In Equation 10 $A_i \cos[(1 + \xi)w_i t + \phi_i]$ can be thought of as the envelope of the carrier, $\cos[(1 + \xi)w_c t]$, which helps to demonstrate that the Doppler shift affects the envelope and the carrier frequency by the same percentage [4]. The Doppler shift also affects the symbol rate and the time synchronization.

3.2. Phase Noise, Oscillator Frequency Offset and Their Effects

The carrier frequency offset has two destructive effects on OFDM systems. The first is the reduction of the amplitude of the desired sub-carrier and the second is the introduction of ICI [6]. The reduction of the amplitude happens because the desired sub-carrier is not sampled at the peak of the sinc function of the DFT since the sinc functions are shifted (see Figure 2). Also, the ICI is caused since orthogonality is lost between the neighbouring sub-carriers [9].

Moose [3] and Pollet *et al.* [2] analytically evaluated the effects of the carrier frequency offset and carrier phase noise on the SNR degradation for an AWGN channel.

The results derived in these papers are used for many of the studies on frequency errors. As a result, we have used these results as benchmarks for the simulation

Moose [3] models the channel by a complex transfer function H_k for the k^{th} sub-carrier and assumes that a frequency offset of ϵ exists for all sub-carriers. The frequency offset, ϵ , is usually expressed relative to the data symbol rate, and it includes both the Doppler shift and the local oscillator frequency offset effects. As mentioned previously, each sub-carrier is affected by the Doppler shift according to its frequency value. Taking all the causes into consideration, the total offset for each sub-carrier after down conversion can be expressed as $\Delta f_i = f_{dc} + f_{di} + \Delta f_{LO}$ where f_{dc} is the carrier Doppler shift, f_{di} is the i^{th} sub-carrier Doppler shift and Δf_{LO} is the local oscillator frequency offset[7].

As can be seen from Equations 8 and 9, $f_{dc} \gg f_{di}$ since $f_c \gg f_i$. Thus,

$\Delta f_i = f_{dc} + \Delta f_{LO} = \Delta f$. This result demonstrates that the frequency offset is independent of the sub-carriers, which in turn states that the relative offset $\epsilon = \Delta f / R_s$ is independent of sub-carriers [4].

The SNR calculated at the output of the DFT of the receiver is given by [3]

$$SNR \geq \frac{E_c}{N_o} \left(\frac{\sin p\epsilon}{p\epsilon} \right)^2 \frac{1}{1 + 0.5947 \left(\frac{E_c}{N_o} \right) (\sin p\epsilon)^2}, |\epsilon| < 0.5 \quad (11)$$

where $\frac{E_c}{N_o}$ is the OFDM carrier energy to the AWGN spectral density. An upper bound on the degradation can be obtained from Equation 11 as follows:

$$D_{dB} \leq 10 \log_{10} \left[\frac{1 + 0.5947 \frac{E_c}{N_o} \sin^2 p\Delta f}{\sin^2 c^2 \Delta f} \right]$$

(12)

where the factor 0.5947 is derived by taking the

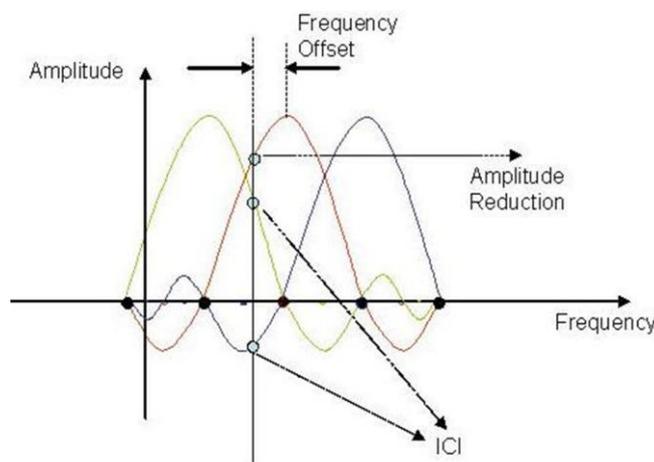


Figure 2: Effects of Frequency Offset: Reduced Amplitude and ICI

purpose.

lower bound of the summation of all interfering sub-carriers [24].

Similar results to Equations 11 and 12 are found in [2] in which the channel is modelled by a time-varying phase $q(t)$, which is a result of either the phase noise of the carriers or a carrier offset between the receiver and transmitter. A distinction is made between phase noise and frequency offset.

3.2.1 Phase Noise

For the phase noise case, $\theta(t)$ is assumed to be a Wiener process with

$$E\{\theta(t)\} = 0 \text{ and } E\{(\theta(t_0 + t) - \theta(t_0))^2\} = 4\pi\beta|t|,$$

where β (Hz) is the one-sided 3-dB linewidth of the Lorentzian power density spectrum of the free-running carrier generator [2].

Phase noise has two main effects. First, it causes a random phase variation common to all sub-carriers. The effects of this common phase error are minimized by employing phase tracking techniques or differential decoding. Second, it introduces ICI. Based on the model defined in [2], the degradation D in SNR, i.e., the required increase in SNR to compensate for the phase noise is

$$D_{dB} \cong \frac{11}{6 \cdot \ln 10} \left(4\pi N \frac{\beta}{R} \right) \frac{E_s}{N_o} \tag{13}$$

Since $R=N/T=NR_s$, where N is the total number of sub-carriers and R_s is the sub-carrier symbol rate. Equation 13 can be rewritten as

$$D_{dB} \cong \frac{11}{6 \cdot \ln 10} \left(4\pi \frac{\beta}{R_s} \right) \frac{E_s}{N_o} \tag{14}$$

The plot of SNR degradation in dB as a function of the normalized bandwidth (β/R_s) is shown in Figure 3

for QPSK. This figure is plotted for $\frac{E_s}{N_o} = 10.5 \text{ dB}$ corresponding to a BER of 10^{-6} for uncoded QPSK [5].

By observing Figure 3, it can be concluded that for a negligible SNR degradation of about 0.1 dB, the -3-dB phase noise bandwidth should be approximately 0.1 percent of the sub-carrier spacing for QPSK. However, this value changes depending on the modulation.

3.2.2 Frequency Offset

For the frequency offset case, $\theta(t)$ is given by

$$\theta(t) = 2\pi\Delta F t + \theta_0 \tag{15}$$

where ΔF is the carrier offset. If there is a frequency offset, then the number of cycles in the FFT interval is not an integer and this causes ICI after the FFT. The sub-carriers in the middle of the OFDM spectrum are more exposed to ICI than the sub-carriers on the edges of the spectrum because they have interfering sub-carriers on their both sides [5].

Based on the model developed in [2], the degradation D in SNR for frequency offset is given by

$$D_{dB} \cong \frac{10}{3 \cdot \ln 10} \left(\pi N \frac{\Delta F}{R} \right)^2 \frac{E_s}{N_o} \tag{16}$$

By using the relation $R = N/T = NR_s$, Equation 16 can be rewritten as

$$D_{dB} \cong \frac{10}{3 \cdot \ln 10} \left(\pi \frac{\Delta F}{R_s} \right)^2 \frac{E_s}{N_o} \tag{17}$$

where $\Delta F/R_s$ is the relative frequency offset. The plot of SNR degradation in dB as function of the relative frequency offset ($\Delta F/R_s$) is shown in Figure 4 for

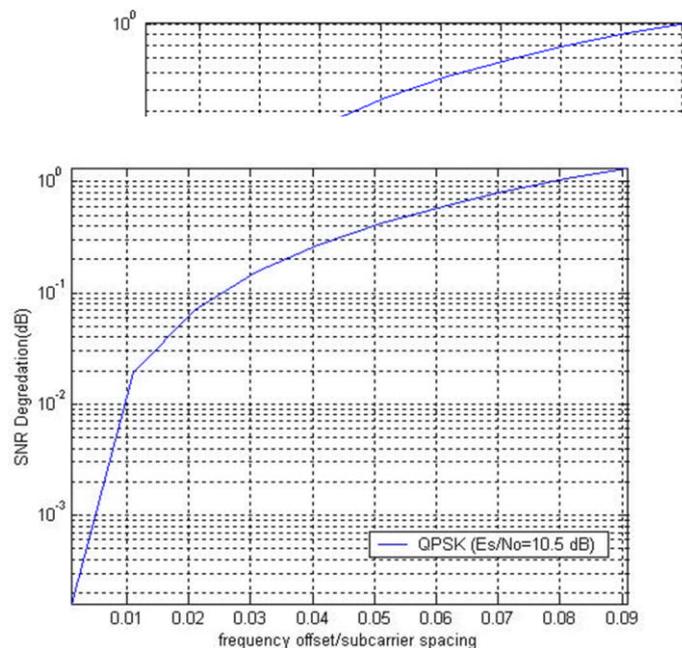


Figure 4: SNR Degradation Versus the Normalized Frequency Offset for QPSK.

QPSK. This figure is plotted for $E_s/N_0=10.5$ dB corresponding to a BER of 10^{-6} for uncoded QPSK [5].

From Figure 4, it can be seen that for a negligible SNR degradation of about 0.1 dB, the tolerable frequency offset should be approximately 2.5% of the sub-carrier spacing for QPSK. However, this value changes depending on the modulation.

4. CONCLUSIONS

This paper discussed the frequency errors and their causes. It was demonstrated that SNR degradation due to the Doppler shift, most of the time, is negligible, and this left frequency offset and phase noise of local oscillators as the main sources of degradation. It was also shown that this degradation varies strongly according to the modulation used. In [6], it was illustrated that 64-QAM could not tolerate more than 1% carrier frequency error for a SNR degradation of 0.5 dB, whereas QPSK modulation could tolerate 5% carrier frequency error for the same SNR degradation. It can also be stated that close spacing of carriers in frequency made the tolerable frequency offset a small percentage of the channel bandwidth. In [3], it is illustrated by simulation that the carrier frequency offset is limited to 4% or less of the inter-carrier spacing to maintain signal-to-interference ratios of 20 dB or greater for the OFDM carriers.

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